Regularized yield criterion and self-consistent estimates for rate-independent plasticity of polycrystals

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Outline

Framework

- Scale transition
- Rate-independent yield criterions

Single crystal plastic behaviour

Homogenization estimates

Results

- Non hardening FCC polycrystal

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Micromechanical modelling approach

- Homogenization of the elastoplastic behaviour of polycrystals

  ✓Overall response (Macroscopic stress-strain curve)

  ✓Statistical description of the local fields (Stress and strain fluctuations)

- Various extensions of the self-consistent model to rate-independent plasticity

  ✓Incipient plastic behaviour (Kröner, 1961)

  ✓Incremental scheme (Hill, 1965; Hutchinson, 1970) and its extensions (e.g. Berveiller and Zaoui, 1979)

  ✓Affine scheme (Masson *et al.* , 2000)

Remark: The field fluctuations within the crystalline orientations are not used to define the elastoplastic tangent modulus.

Standard Schmid law

Let $\mathcal{S}$ the set of slip systems of the single crystal.

The plastic yield surface is defined by a multi-criterion

$$\boxed{\sup_{k\in\mathcal{S}} f^k(\boldsymbol{\sigma}) = 0} \quad \text{with} \quad f^k(\boldsymbol{\sigma}) = \boldsymbol{\mu}^k : \boldsymbol{\sigma} - \tau_c^k$$

Drawbacks:

- Sharp corners $\Rightarrow$ direction of flow is not uniquely defined
- Several sets of slip systems may exist (selection criterion)
- Ill-conditioned problem for some hardening laws

Regularized Schmid law

The standard Schmid law is approximated by a single criterion

$$f(\boldsymbol{\sigma}) = 0$$

Various proposals for f (see, for instance, Montheillet *et al.* ,1985; Gambin,1991,1992; Darrieulat and Piot, 1996; Franz *et al.* ,2009...)

Common features:

- Yield surface with rounded corners
- Unique direction of plastic flow with the normality rule
- All slip systems are active
- No restrictions on the hardening description

Gambin's regularization

The regularized yield function reads

$$f(\boldsymbol{\sigma}) = \left\{ \sum_k \left(\frac{\tau^k}{\tau_c^k} \right)^{2n} \right\}^{1/2n} - 1$$

- f is differentiable and strictly convex
- The regularized yield surface is arbitrarily close to the Schmid one when $n \longrightarrow \infty$
- By contrast with the Schmid law, the slip rates on each slip system k derive from an unique plastic multiplier $\dot{\lambda}$

$$\dot{\gamma}^k = \dot{\lambda} \frac{\partial f}{\partial \tau^k}$$

- $\dot{\lambda}$ is determined with the consistency condition

Problems adressed

- Implications of the regularization on the plastic behaviour under uniaxial tension

  ✓at the single crystal level

  ✓at the polycrystal level

- Is there a dependence on the chosen scale transition scheme ?
- How does the self-consistent estimates compare with available bounds on the effective strength of the polycrystalline aggregate ?

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Description of the plastic behaviour

For sake of simplicity, perfect plasticity is considered.

The plastic response is described using the tangent constitutive relation

$$\dot{\boldsymbol{\sigma}} = \mathbf{L} : \dot{\boldsymbol{\varepsilon}}.$$

Elastoplastic tangent modulus $\mathbf{L}$

- Schmid law $\Rightarrow \mathbf{L} = \mathbf{C} - \sum_k \left\{ \mathbf{C} : \boldsymbol{\mu}^k \otimes \sum_{k'} Y^{kk'} \boldsymbol{\mu}^{k'} : \mathbf{C} \right\}, \; Y^{kk'} = [\boldsymbol{\mu}^k : \mathbf{C} : \boldsymbol{\mu}^{k'}]^{-1}$
- Gambin law $\Rightarrow \mathbf{L} = \mathbf{C} - \dfrac{\mathbf{C} : (\mathbf{G} \otimes \mathbf{G}) : \mathbf{C}}{\mathbf{G} : \mathbf{C} : \mathbf{G}}, \; \mathbf{G} = \sum_k \dfrac{\boldsymbol{\mu}^k}{\tau_c^k} \left(\dfrac{\tau^k}{\tau_c^k} \right)^{2n-1}$

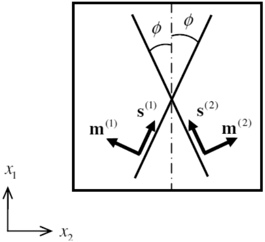
With $n \rightarrow \infty$: same yield locus and active slip systems but different elasto-plastic tangent modulus, in general.

Single slip case

In the limit $n \to \infty$, the regularized elastoplastic tangent modulus coincides with the one derived from the Schmid law

$$\mathbf{L} = \mathbf{C} - \frac{\mathbf{C} : (\boldsymbol{\mu}^1 \otimes \boldsymbol{\mu}^1) : \mathbf{C}}{\boldsymbol{\mu}^1 : \mathbf{C} : \boldsymbol{\mu}^1}$$

Double slip case



- The components of the tangent modulus coincide except

$$L_{1212} = \begin{cases} 0 & \text{for Schmid law,} \\ C_{1212} & \text{for regularized law} \end{cases}$$

- The Schmid law leads to a reduced shear tangent modulus in general.

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Self-consistent scheme

The investigation is focused on the Hill's incremental self-consistent model.

- Constitutive equation

$$\langle\dot{\boldsymbol{\sigma}}\rangle_r = \mathbf{L}_r : \langle\dot{\boldsymbol{\varepsilon}}\rangle_r\,, \quad \mathbf{L}_r \text{ uniform within phase } r$$

- Localization relation

$$\langle\dot{\boldsymbol{\varepsilon}}\rangle_r = \mathbf{A}_r : \dot{\bar{\boldsymbol{\varepsilon}}}, \quad \mathbf{A}_r = \left\{\mathbf{I} + \mathbf{P} : (\mathbf{L}_r - \widetilde{\mathbf{L}})\right\}^{-1}$$

with $\mathbf{P}$ the Hill tensor.

- Effective behaviour

$$\dot{\bar{\boldsymbol{\sigma}}} = \widetilde{\mathbf{L}} : \dot{\bar{\boldsymbol{\varepsilon}}} \quad \text{with} \quad \widetilde{\mathbf{L}} = \langle \mathbf{L}_r : \mathbf{A}_r \rangle$$

Taylor model as a particular case

$$\mathbf{A}_r = \mathbf{I} \Longrightarrow \widetilde{\mathbf{L}} = \langle \mathbf{L}_r \rangle$$

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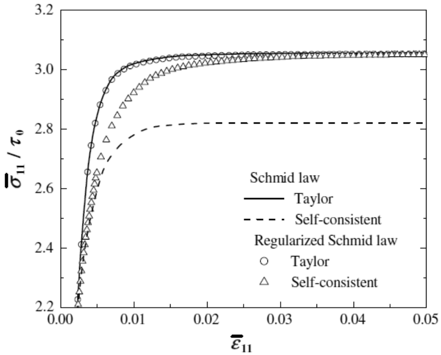
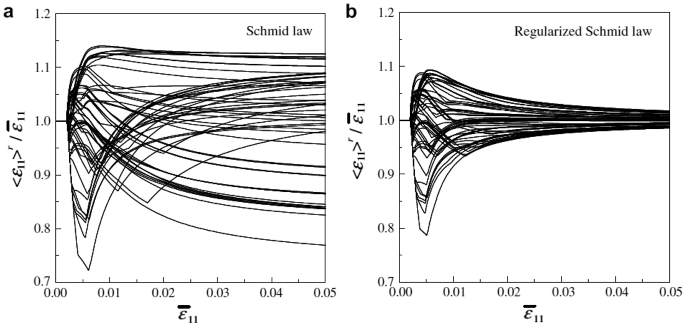
Overall response



- Taylor estimates are insensitive to the yield criterion.
- The self-consistent model predicts an ealier plastic yielding.
- The SC estimate of the flow stress strongly depends on the yield criterion.
- SC model with the regularized criterion $\Rightarrow$ flow stress $\simeq$ Taylor estimate

Fluctuation of the average local strain field

Self-consistent estimates



- The description of the strain heterogeneity depends on the yield criterion
- Regularization leads to very low strain fluctuations in the plastic regime
- Local description in agreement with the macroscopic behaviour

Discussion

Reduced tangent modulus and **P** tensor

- The **P** tensor reflects the difference between local and effective tangent moduli to the localization tensor

$$\mathbf{A}_r = \left\{ \mathbf{I} + \mathbf{P} : (\mathbf{L}_r - \widetilde{\mathbf{L}}) \right\}^{-1}$$

- The higher the components of **P**, the more enhanced the strain heterogeneity
- **P** is a decreasing function of the effective tangent modulus $\widetilde{\mathbf{L}}$
- The Schmid law which gives a reduced shear tangent modulus thus leads to the highest strain heterogeneity

Discussion

Link with power-law viscoplastic regularization

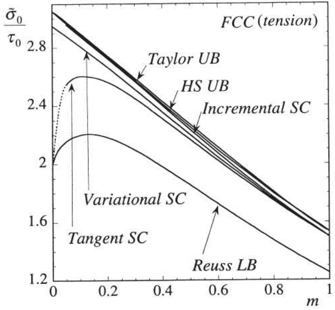


Figure taken from (Nebozhyn, Gilormini, Ponte Castañeda, JMPS, 2001)

- SC flow stress with Gambin law coincides with the viscoplastic one in the rate-independent limit (~ 3.06)
- SC flow stress with Schmid law (2.82) respects the rigorous variational self-consistent estimate for rigid-plastic behaviour (2.948)

Summary

- Self-consistent estimates for rate-independent plasticity are highly sensitive to the description of the single crystal yield criterion
- Gambin regularized criterion leads to an effective flow stress close to the Taylor model and a very low strain heterogeneity
- It is conjectured that any regularization would present the same shortcomings
- Reduced shear tangent modulus is responsible for the wide strain heterogeneity predicted with the Schmid law
- The flow stress obtained with the Schmid law (Hutchinson, 1970) does not violate the variational self-consistent estimate of Ponte Castañeda

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