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The modified affine homogenisation scheme for nonlinear composites : comparison with variational bounds and estimates

R. Brenner, O. Castelnau

LPMTM, Université Paris XIII, Villetaneuse, France

P. Gilormini

LMT, ENS Cachan, Université Paris VI, Cachan, France

General framework

➡ Deductive description of the **effective behaviour** and the **local mechanical fields** of a nonlinear heterogeneous material.

Random media

Partial statistical description
of the microstructure
+
Linear constitutive behaviour
(uniform per phase)

➡ **Homogenisation approach**

Nonlinear behaviour

Constitutive behaviour depends on the mechanical fields !

➡ How to build a **nonlinear homogenisation scheme** ?

Choice of a **Reference field per phase** **+** **Linearisation procedure**

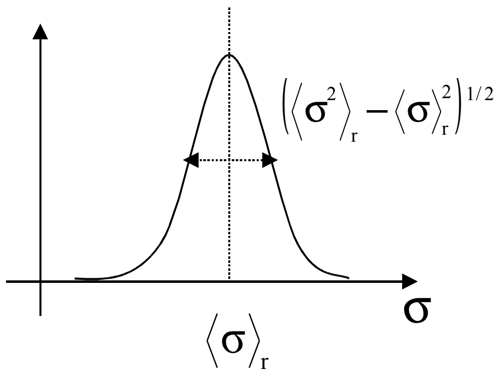
Classical results of linear homogenisation

Constitutive behaviour : $\varepsilon(x) = s_r : \sigma(x) + \varepsilon_r^0 \quad \forall x \in \Omega_r$

Compliance — Eigenstrain

Homogeneous macroscopic stress : Σ

★ Statistical description of the mechanical fields



Averaged localisation : $\langle\sigma(x)\rangle_r = \langle B(x)\rangle_r : \Sigma + \langle\sigma_{res}(x)\rangle_r$

$\langle B\rangle_r$ tensor obtained by using Eshelby's solution of the inclusion problem

Intraphase second moment :

$$\langle\sigma \otimes \sigma\rangle_r = \frac{1}{c_r}\left[(\Sigma \otimes \Sigma) :: \frac{\partial S}{\partial s_r} + \left\langle \varepsilon_s^0 : \frac{\partial}{\partial s_r}\left(2\langle\sigma\rangle_s - \langle\sigma_{res}\rangle_s\right)\right\rangle\right]$$

★ Effective behaviour

$$E = S : \Sigma + E^0$$

$$S = \langle s_r : B_r \rangle \qquad E^0 = \langle {}^tB_r : \varepsilon_r^0 \rangle$$

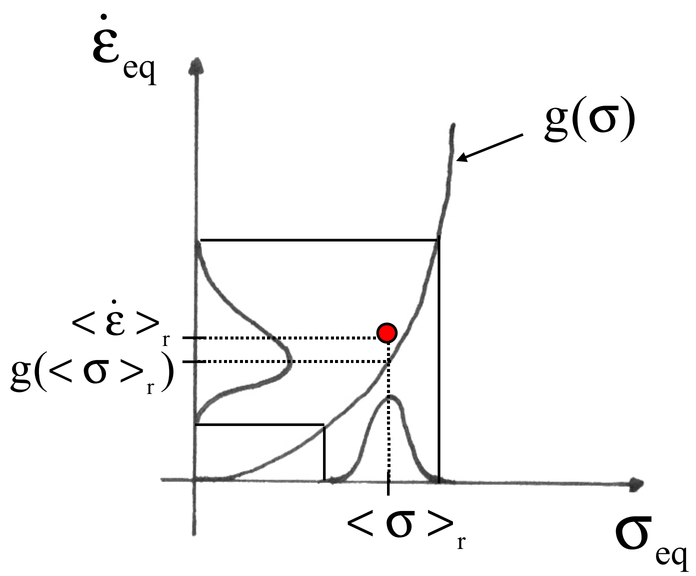
Nonlinear behaviour and intraphase heterogeneity

Intraphase heterogeneity of the mechanical fields

→ Non uniformity per phase of the behaviour

→ Average fields $<\sigma>_r$ and $<\dot{\varepsilon}>_r$ not linked by the constitutive relation

Schematic representation



Generally,

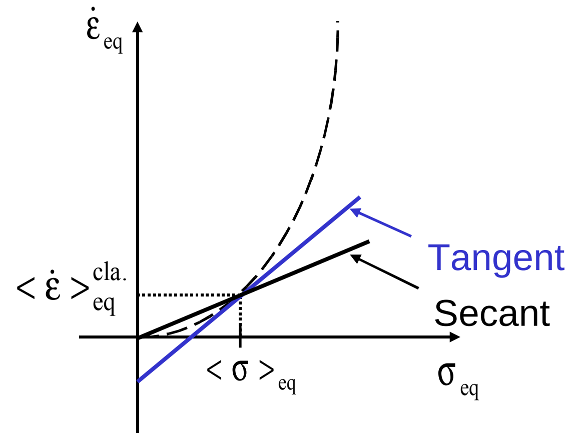
$$<\dot{\varepsilon}>_r \neq g(<\sigma>_r)$$

Some previous propositions...

★ "Classical" estimates

Reference field = Average per phase

$$<\dot{\varepsilon}>_r = g(<\sigma>_r)$$



- Secant model (Berveiller and Zaoui 1979)

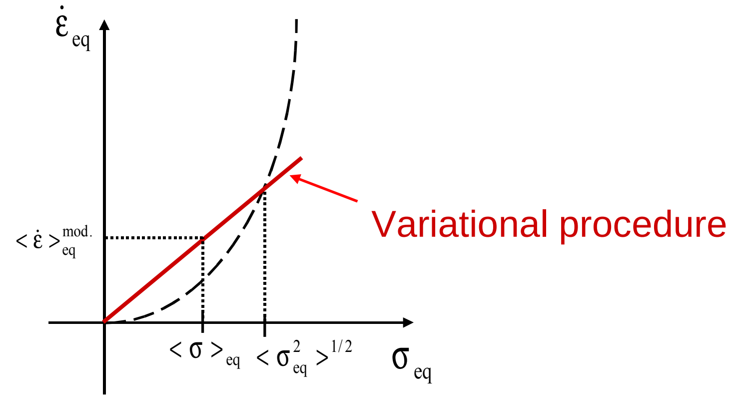
- Incremental model (Hill 1965, Hutchinson 1970)
- Tangent model (Molinari et al. 1987)
- Affine model (Masson et al. 1998)

★ Variational bound (Ponte Castaneda, 1991)

Reference field = Second moment per phase

(Suquet 1995)

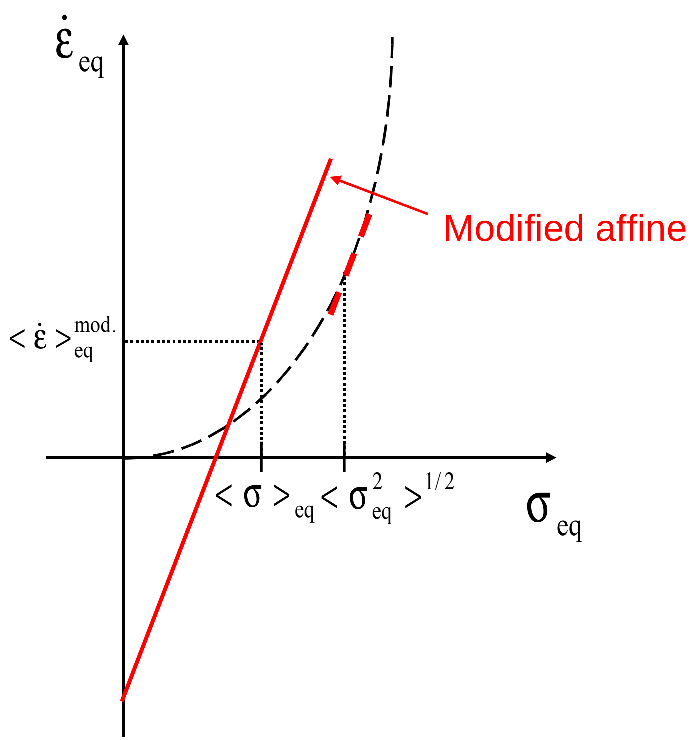
$$<\dot{\varepsilon}>_r \neq g(<\sigma>_r)$$



It delivers rigorous nonlinear upper bound for the effective energy

Linearisation

Reference field	Tangent	Secant
$< \sigma >_{eq}$	- Tangent model (1987) - Affine model (1998)	- Secant model (1979)
$< \sigma^2_{eq} >^{1/2}$	- Modified affine	- Variational procedure (1991)



"Thermoelastic" linearised behaviour

$$\langle \dot{\varepsilon} \rangle_r = m^r : \langle \sigma \rangle_r + \varepsilon^0_r$$

m^r and ε^0_r defined with respect to $\langle \sigma \otimes \sigma \rangle_r$

Application : two-phase isotropic incompressible composite

★ Local behaviour $$\frac{\sigma_{eq}}{\sigma_0} = \left(\frac{\dot{\varepsilon}_{eq}}{\dot{\varepsilon}_0}\right)^m$$ Overall behaviour $$\frac{\Sigma_{eq}}{\Sigma_0} = \left(\frac{\dot{E}_{eq}}{\dot{E}_0}\right)^m$$

m Strain rate sensitivity exponent

σ_0 Reference stress

Contrast $\sigma_0^2 / \sigma_0^1 = 10$

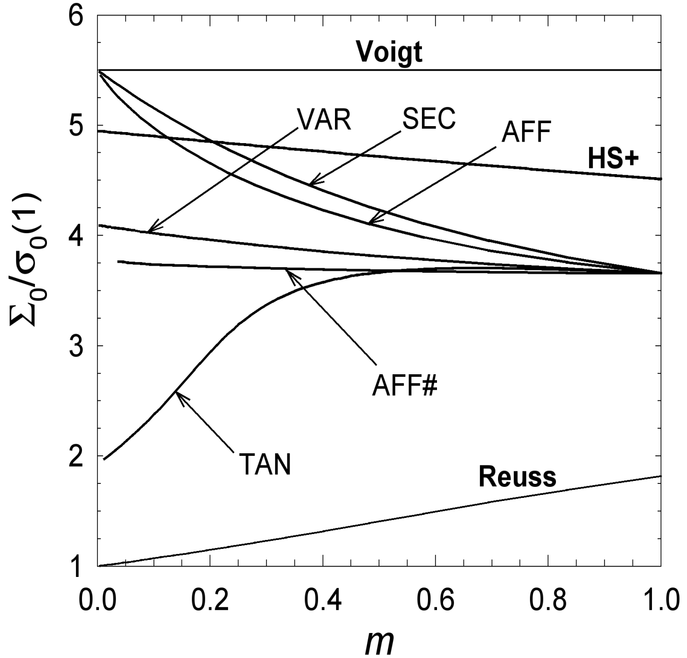
Volume fraction 50%

★ Uniaxial tensile loading

★ Self-consistent type microstructure

Overall behaviour

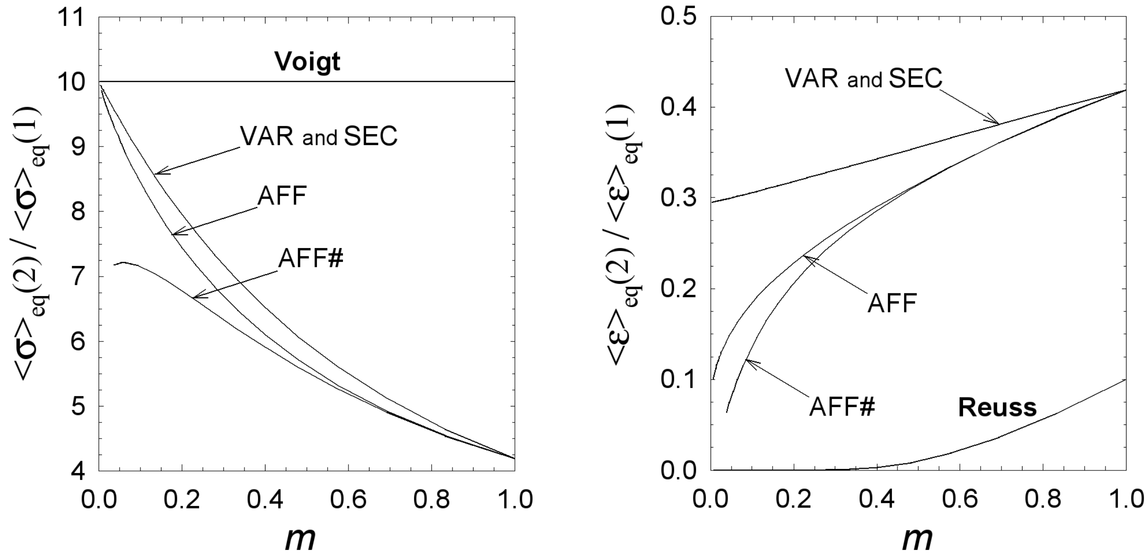
Reference stress Σ_o



- Affine and secant models violates HS+ bound
- Tangent model tends to Reuss
- Variational estimate respect HS+ bound (by construction)
- Modified affine respect both HS+ bound and variational estimate

Quantitative improvement on classical estimates

Interphase fields heterogeneity



- Variational estimate = Secant estimate (Gilormini et al. 2001)

No trivial link between effective behaviour and local fields !

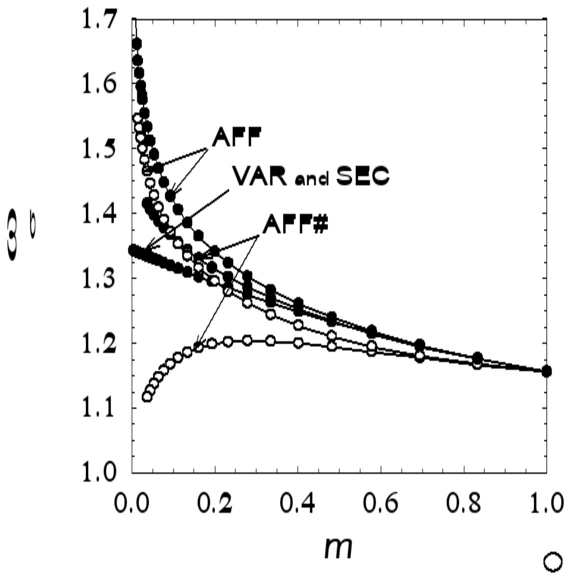
- Modified affine predicts the highest interphase strain heterogeneity and the lowest interphase stress heterogeneity

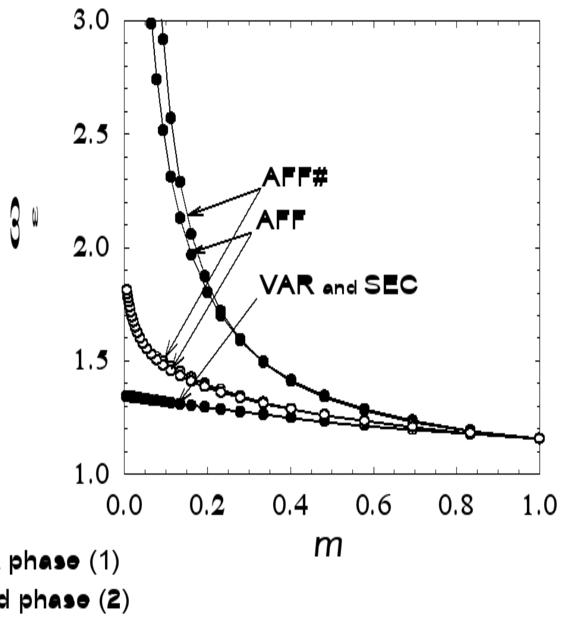
Qualitative improvement on former theories

Intraphase fields heterogeneity

Heterogeneity ratios

$$\omega_\sigma = \sqrt{\frac{\langle \boldsymbol{\sigma} : \boldsymbol{\sigma} \rangle_r}{\langle \boldsymbol{\sigma} \rangle_r : \langle \boldsymbol{\sigma} \rangle_r}} \qquad \omega_\varepsilon = \sqrt{\frac{\langle \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} \rangle_r}{\langle \boldsymbol{\varepsilon} \rangle_r : \langle \boldsymbol{\varepsilon} \rangle_r}}$$





- Secant-type estimates predict the same heterogeneity in both phases
- Affine-type estimates predict a stronger heterogeneity in the "hard" phase

Conclusion

- An accurate nonlinear homogenisation procedure has to take into account the intraphase field heterogeneity.

- The modified affine procedure is a simple way to do it.
  - ➡ It makes use of a virtual linear "thermoelastic" medium incorporating the second moment of the intraphase fields.
  - ➡ Improved estimates at both local and global scales.
  - ➡ Not restricted to behaviours deriving from a potential.

Perspectives

- Extension to elastoviscoplastic behaviours (no approach of this type at present)

- Application to polycrystals (composites with large number of anisotropic phases)

- Confrontation of the different models to local statistical measurements of the mechanical fields (diffraction for instance)

- Use of the second moment to predict microstructural evolutions : damage, phase transformation, texture evolution etc...